Kittel 2.2(a)(b)

2.2(a) The volume is given by $|\vec{a}_1 \cdot \vec{a}_2 \times \vec{a}_3|$.

$$\vec{a}_2 \times \vec{a}_3 = -c\left(\frac{3^{1/2}a}{2}\right)(\hat{x}\times\hat{z}) + c\left(\frac{a}{2}\right)(\hat{y}\times\hat{z})$$

$$= +c\frac{3^{1/2}}{2}a\,\hat{y} + \frac{ca}{2}\hat{x}.$$

$$\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) = \frac{ca^{1/2}}{2}a\left(\frac{a}{2}\right)\times 2 =$$

$$3^{1/2}\frac{a}{2}\left(\frac{ca}{2}\right) + \left(\frac{a}{2}\right)\left(c\frac{3^{1/2}}{2}a\right)$$

$$= \frac{ca^2}{4}3^{1/2} + \frac{ca^2 3^{1/2}}{4} = \boxed{\frac{ca^2}{2}3^{1/2}}$$

(b) Let $A = \frac{3^{1/2}a}{2}$, $B = \frac{a}{2}$.

$$a_1 = A\hat{x} + B\hat{y}, \quad a_2 = -A\hat{x} + B\hat{y}, \quad a_3 = c\hat{z}.$$

$$b_1 = 2\pi\frac{a_2 \times a_3}{a_1 \cdot (a_2 \times a_3)} = 2\pi\frac{Ac\hat{y} + Bc\hat{x}}{2ABc} = \boxed{\pi\left[\frac{\hat{y}}{B} + \frac{\hat{x}}{A}\right]}$$

$$b_2 = 2\pi\frac{a_3 \times a_1}{a_2 \cdot (a_2 \times a_3)} = 2\pi\frac{Ac\hat{y} - Bc\hat{x}}{2ABc} = \boxed{\pi\left[\frac{\hat{y}}{B} - \frac{\hat{x}}{A}\right]}$$

$$b_3 = 2\pi\frac{a_1 \times a_2}{a_1 \cdot (a_2 \times a_2)} = 2\pi\cdot\frac{AB\hat{z} + AB\hat{z}}{2ABc} = \frac{2AB\hat{z}}{2ABc}(2\pi)$$

$$= \boxed{\left(\frac{2\pi}{c}\right)\hat{z}}$$